IIR filter design by Impulse Invariant Method

By uniformly sampling the impulse response of equivalent analog filter, the impulse response of digital IIR filter is obtained.

i.e. $h(n) = h_a(nT)$ — (1)

T = sampling period.

Consider the simple distinct pole case first, the transfer function of analog filter is

$$H_a(S) = \sum_{i=1}^{M} \frac{A_i}{S-p_i} \quad \text{— (2)}$$

Taking inverse Laplace Transform of above

$$h_a(t) = \sum_{i=1}^{M} A_i e^{p_i(t)} u_a(t)$$

∴ As per eq (1), the impulse response of digital filter will be

$$h(n) = h_a(nT) = \sum_{i=1}^{M} A_i e^{p_i \cdot nT} u_a(nT)$$

Taking z-Transform of above

i.e. $$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

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$$\therefore\ H(z) = \sum_{n=0}^{\infty}\left[\sum_{i=1}^{M} A_i\, e^{p_i nT}\, u_a(nT)\right] z^{-n}$$

Interchange order of summation

$$H(z) = \sum_{i=1}^{M}\left[\sum_{n=0}^{\infty} A_i\, e^{p_i nT}\, u_a(nT)\right] z^{-n}$$

$$\therefore\ H(z) = \sum_{i=1}^{M} \frac{A_i}{1 - e^{p_i T} z^{-1}} \quad \text{--- (3)}$$

using eq. (2) & (3)

The mapping formula is

$$\frac{1}{s - p_i} \rightarrow \frac{1}{1 - e^{p_i T} z^{-1}}$$

$\therefore$ analog pole at $s = p_i$ is mapped to digital pole at $z = e^{p_i T}$

$\therefore$ analog pole & digital poles are related as

$$z = e^{sT} \quad \text{--- (4)}$$

If $s = \sigma + j\Omega$

and $z = re^{j\omega}$

eq. (4) becomes

$$re^{j\omega} = e^{\sigma T} \cdot e^{j\Omega T}$$

$$\therefore \quad r = e^{\sigma T}$$

$$\omega = \Omega T$$

$\Rightarrow$ if $\sigma < 0$ then $0 < r < 1$

$\sigma > 0$ then $r > 1$

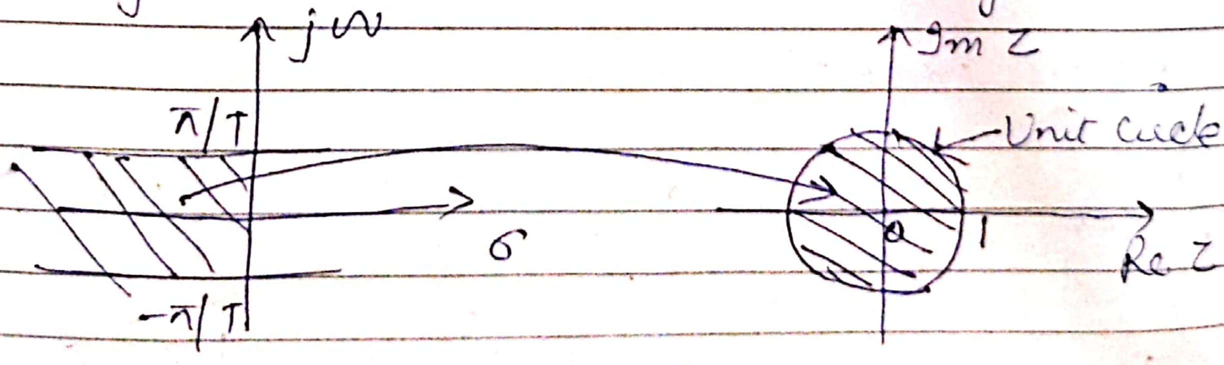
$\sigma = 0$ then $r = 1$

$\therefore$ left half of s-plane is mapped inside unit circle in z plane

right half of s-plane is mapped outside unit circle in z-plane.

which is desirable property of stability

and $j\Omega$ axis maps from many to one



s-plane (analog domain) z-plane (digital domain)

Mapping of $z = e^{sT}$

How the mapping is many to one?

1. $\omega = \Omega T \Rightarrow$ interval $-\frac{\pi}{T} \leq \Omega \leq \frac{\pi}{T}$

maps into values $-\pi \leq \omega \leq \pi$

2. also, interval $\frac{\pi}{T} \leq \Omega \leq \frac{3\pi}{T}$ will map

into $-\pi \leq \omega \leq \pi$

3. In general, interval $(2K-1)\frac{\pi}{T} \leq \Omega \leq (2K+1)\frac{\pi}{T}$

will map into $-\pi \leq \omega \leq \pi$,

$\therefore$ mapping from analog freq Ω to frequency variable ω in digital domain is many to one which will result in aliasing due to sampling of impulse response.

Other Properties are.

$$\frac{1}{(s+s_i)^m} \rightarrow \frac{(-1)^{m-1}}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left[\frac{1}{1-e^{-sT}z^{-1}}\right]; \; s \to s_i$$

$$\frac{s+a}{(s+a)^2+b^2} \rightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \rightarrow \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

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Example 8.5 For the analog transfer function

$$H(s) = \frac{1}{(s+1)(s+2)}$$ Laplace Tran.

determine $H(z)$ using impulse invariant technique. Assume $T = 1s$.

Solution Using partial fractions, $H(s)$ can be written as

$$H(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{A(s+2) + B(s+1)}{(s+1)(s+2)}$$

$$1 = A(s+2) + B(s+1)$$

Letting $s = -2$ we get $B = -1$ and letting $s = -1$, we get $A = 1$. Therefore

$$H(s) = \frac{1}{s+1} - \frac{1}{s+2}$$ L.T. Poles at P_i = $-1, -2$

The system function of the digital filter is obtained by using Eq. 8.23.

$$H(z) = \frac{1}{1 - e^{-T} z^{-1}} - \frac{1}{1 - e^{-2T} z^{-1}}$$ $\frac{1}{1 - e^{P_i t} z^{-1}}$

$$= \frac{z^{-1} [e^{-T} - e^{-2T}]}{1 - (e^{-T} + e^{-2T}) z^{-1} + e^{-3T} z^{-2}}$$

Since $T = 1s$,

$$H(z) = \frac{0.2326 z^{-1}}{1 - 0.5032 z^{-1} + 0.0498 z^{-2}}$$

Example 8.4 Convert the analog filter into a digital filter whose system function is

$$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$$

$$H(s)=\frac{s+0.2}{(s+0.2)^2+9}$$

Use the impulse invariant technique. Assume $T = 1$s.

Solution The system response of the analog filter is of the standard form

$$H(s)=\frac{s+a}{(s+a)^2+b^2}$$

where $a = 0.2$ and $b = 3$. The system response of the digital filter can be obtained using Eq. 8.27.

$$H(z)=\frac{1-e^{-aT}(\cos bT)\,z^{-1}}{1-2e^{-aT}(\cos bT)\,z^{-1}+e^{-2aT}\,z^{-2}}$$

$$=\frac{1-e^{-0.2T}(\cos 3T)\,z^{-1}}{1-2e^{-0.2T}(\cos 3T)\,z^{-1}+e^{-0.4T}\,z^{-2}}$$

Taking $T = 1$s,

$$H(z)=\frac{1-(0.8187)(-0.99)\,z^{-1}}{1-2(0.8187)(-0.99)\,z^{-1}+0.6703\,z^{-2}}$$

That is,

$$H(z)=\frac{1+(0.8105)\,z^{-1}}{1+1.6210\,z^{-1}+0.6703\,z^{-2}}$$